On supply and demand

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Abstract

Supply and demand plots are not continuous because price and quantity are not infinitely divisible. They are discrete variables. Real supply and demand plots, if they were to be derived from real world empirical observations, would be a set of discrete points not continuous lines. Real supply and demand plots are also, self evidently, time dependent.

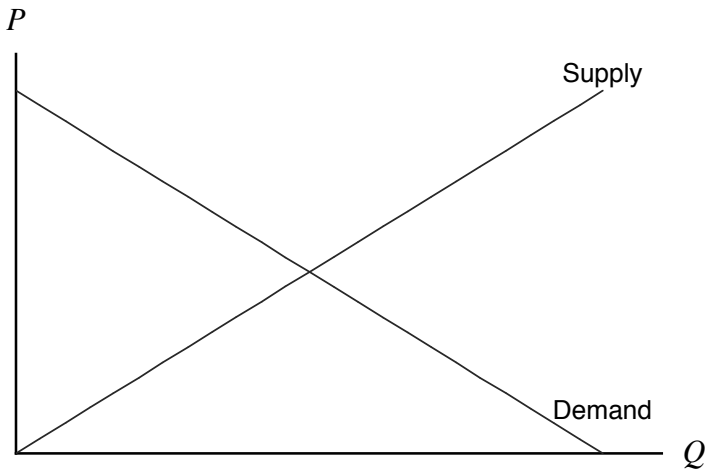

Fig. 1 A "supply and demand" plot of the type often found in undergraduate economics textbooks.

Economic textbooks present pictures of how supply and demand relate to price similar to that shown in Fig. 1. If a student were to ask a finance lecturer for data to support a particular model, lets say the log-normal distribution assumption underpinning Black-Scholes option pricing, data would be forthcoming and, to a first approximation, it would fit the model. If a student were to ask an economics lecturer to see supply and demand curves for any consumer good that you can think of (chocolate bars, motor vehicles, coffee, paper clips, bananas …) they would inevitably be greeted with a blank stare. Do supply and demand curves for real life consumer goods derived from real life empirical data exist?

Supply and demand figures like the one shown in Fig. 1, which is typical of the way these are portrayed in undergraduate texts, are not derived from empirical observation. But suppose they could be; what would they look like? What would the supply and demand curves for iPods look like? The mention of iPods raising an interesting point about these plots. They say nothing about changes in time. Time is absent. A supply curve and demand curve for a particular good has presumably always existed and always will exist. No? Then why the absence of time as a variable? Common sense, let alone observation of the real world tells us that demand for iPods in 1905 was different to demand in 2005. Likewise supply differed. Clearly both supply and demand are things that change over time yet there is no mechanism within the textbook model by which these supply and demand lines can change.

Whereas a 101 physics textbook may describe how a trajectory of a ball changes over time the 101 level economics textbook models do not tell us how a supply and demand "trajectory" could occur. Similarly macroeconomics talks of short run and long run outcomes. Again, in this two state world, no mechanism is offered in textbooks to describe the transition between these two states. Since short run and long run refer to periods in time a mathematical description of how the transition from short term to long run takes place must include time. Equations should exist that lead to the simplified "long run" case as $t \to \infty$.

In the real world both supply and demand are time dependent, for example the demand for iPods today is different today from last month, last year, 4 years ago and so on. So at the very least we need to add a third axis to supply and demand plots to represent time (Fig. 2).

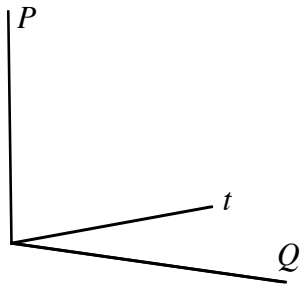

Fig. 2 Real supply and demand depends not just on price P but also on time t.

Including a third axis allows the inclusion of time and therefore would allow the market dynamics to be measured – again, assuming that these plots could be derived from empirical observation – and plotted.

So rather than looking like Fig. 1 one might expect real supply and demand curves to look like Fig. 3 below, changing over time in some way. This means that the intersection of the demand and supply lines/curves should be time dependent. By definition is something changes with time it cannot be in equilibrium.

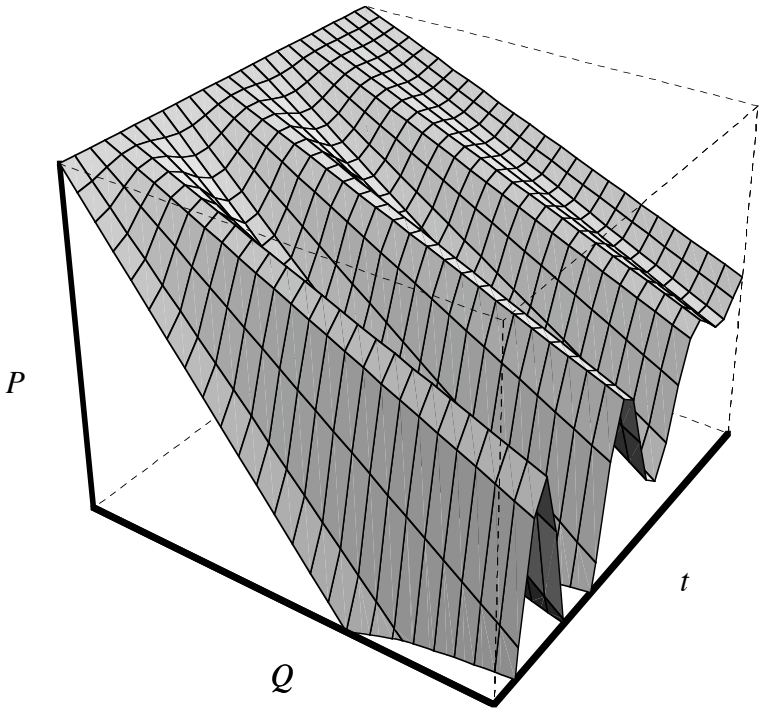

Fig. 3 A hypothetical continuous time dependent demand curve.

A plot such as that shown in Fig. 1 would therefore be meaningless unless accompanied by information stating the specific date at which the data plotted in that snapshot in time were recorded.

The next question to ask is if someone were to produce Fig. 1 and tell you that it represents the supply and demand for iPods in the USA on July 3rd 2005 would that be acceptable (notwithstanding the minimal labelling of *my* Fig. 1)?

Or asking the question in a more general way: in Fig. 1 suppose for a moment we could suspend our disbelief and assume that time did not exist. Under this condition would the figure be valid? The figure shows supply and demand lines (curves) that are presumed to be continuous. This assumption, it turns out, is due to the fact that these lines are ultimately postulated from utility theory in which it is assumed that utility is continuous. Of course utility theory has its own mathematical baggage [1].

The idea that both supply and demand are represented as continuous lines implies that both price and quantity are infinity divisible. If such a world were to exists a car producer may produce and sell 87.456234 cars, a chocolate bar maker may produce and sell 937.23954367 chocolate bars. The consumer may be asked to pay $1.2034923456783 for each chocolate bar. Such a world does not exist. Both price and quantity have a minimum unit, they are both quantized. Producers of goods supply integer quantities of those goods. The car producer supplies 87 cars or 88 cars but not 87.456234 cars. Likewise suppliers of services will charge their labour in a minimum unit. It may be per hour or per 15 minutes or per 5 minutes but a minimum quantum exists. A minimum quantum of price exists. Price is quantized. The consumer purchasing a chocolate bar will pay an amount rounded (up or down) to the nearest minimum unit of price in that market. For example if the consumer were to purchase a single chocolate bar in Australia they would pay in multiples of 5 cents.

The use of lines, whether straight or curved, in textbook supply and demand plots is incorrect and

misleading. In fact the price and quantity axes are divided into multiples of these minimum units and thus the supply and demand curves are not actually lines or curves but really a series of points (Fig. 4). Nothing exists or can exist between the points. Supply and demand are discrete *not* continuous.

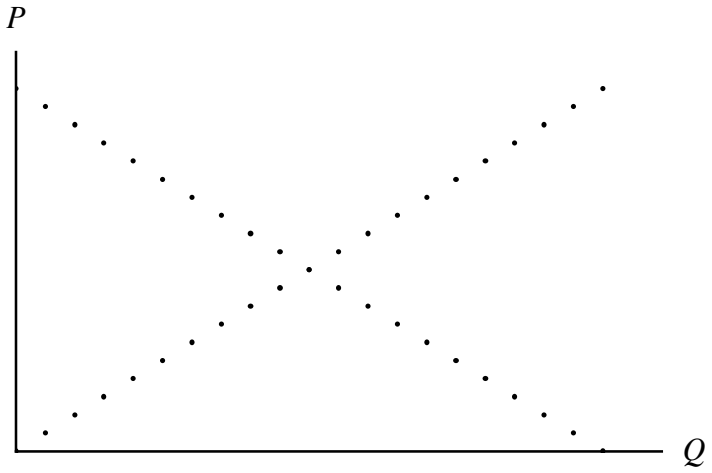

Fig. 4 A real "supply and demand" plot, at a fixed point in time, would be a set of discrete points not continuous lines.

Joining the dots may be a useful game for small children however for those wanting to understand the real world joining the dots in Fig. 4 in order to produce something resembling Fig. 1 is invalid because nothing exists between the dots. Producers are not supplying fractions of goods and services. Consumers are not paying fractions of cents per (integer) item. To join the dots and thus imply that price and quantity vary smoothly and continuously is wrong and misleading because neither price nor quantity are infinitely divisible. While we may or may not be able to construct price versus quantity plots of supply and demand that have been derived from real world empirical data what we know for a fact is that in the real world price and quantity are both discrete.

The science of astronomy did not evolve with astronomers getting a good nights sleep and then spending the days developing axioms and postulating how the stars and planets *should* move in the night skies. Astronomers made empirical observation of the night skies and developed models to explain those observations. One would have thought that the concept of supply and demand should have similarly evolved from real world empirical observations rather than axioms and postulates. Open your office windows and have a look outside. There is a complex dynamic (time dependent and ever changing) world out there.

References

[1] J. L. McCauley, "The futility of utility: how market dynamics marginalize Adam Smith", *Physica A*, **285** (2000) 506.

